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Single-Ended and Differential S-Parameters

Maxim Integrated Products

Single-Ended and Differential S-Parameters

1 Overview

Differential circuits have been important in communication systems for many years. In the past, differential communication circuits operated at low frequencies, where they could be designed and analyzed using lumped-element models and techniques. With the frequency of operation increasing beyond 1GHz, and above 1Gbps for digital communications, this lumped-element approach is no longer valid, because the physical size of the circuit approaches the size of a wavelength**.**

Distributed models and analysis techniques are now used instead of lumped-element techniques. Scattering parameters, or S-parameters, have been developed for this purpose [1]. These S-parameters are defined for single-ended networks. S-parameters can be used to describe differential networks, but a strict definition was not developed until Bockelman and others addressed this issue [2]. Bockelman's work also included a study on how to adapt single-ended S-parameters for use with differential circuits [2]. This adaptation, called "mixed-mode S-parameters," addresses differential and common-mode operation, as well as the conversion between the two modes of operation.

This application note will explain the use of single-ended and mixed-mode S-parameters, and the basic concepts of microwave measurement calibration.

2 Single-Ended S-Parameters

The term "scattering parameters" is derived from the parameters that represent a scattering or separation of a signal by a device under test (DUT). These scattered signals are the reflected and transmitted waves that are produced when a device is struck with an incident wave. S-parameters become important when the operating frequencies are high enough so that circuit elements become a significant fraction of a wavelength (approximately one-tenth of a wavelength), and a lumped-element approach must be discarded in favor of a distributed model.

Also, when the frequency increases to the microwave range, it is difficult to measure voltages and currents as required for impedance measurements. To overcome this problem, a ratio of the incident and the outgoing wave is used. This is represented in Figure 1 and defined in equation (1) [1].

$$S_{ij} = \left. \frac{b_i}{a_j} \right|_{a_k = 0 \ for \ k \neq j} \qquad (1)$$

Equation (1) states that, to measure S_{ij}, energize port j and measure the response on port i. It is important to note that all ports, except the stimulus port, must be terminated with that port's characteristic impedance. For example, to calculate S_{21} for the network in Figure 1, energize port 1, take the power out of port 2, and divide it by the power incident on port 1 when there are no incident voltages on port 2. To achieve this condition of no incident voltage on port 2, terminate port 2 with its characteristic impedance (typically 50Ω).

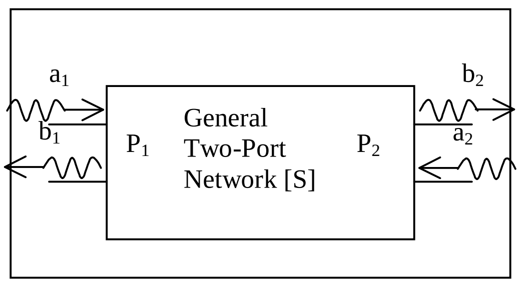

Figure 1. Example of a two-port network

In matrix form, equation (1) becomes equation (2):

$$[b]=[S][a] \qquad (2)$$

In equation (2), [b] is an n×1 column matrix, [a] is an n×1 column matrix, and [S] is an n×n matrix, where n is the number of ports in the network (that is, n = 2 in Figure 1).

The power waves are related to the voltages and currents by the following equations [1]:

$$a_n = \frac{v_n + i_n Z_n}{2\sqrt{\mathrm{Re}(Z_n)}} \qquad (3)$$

$$b_n = \frac{v_n - i_n Z_n^*}{2\sqrt{\mathrm{Re}(Z_n)}} \qquad (4)$$

In equations (3) and (4), v_n is the total voltage at port n, i_n is the total current at port n, and Z_n is the characteristic impedance at port n.

S-parameters are used in many ways to characterize a device or transmission line. In the [S] matrix, the diagonal elements (S_{ii}) are the reflection coefficients if, and only if, all other ports are terminated with their characteristic impedance. The voltage standing-wave ratio (VSWR), the return loss (in dB), and other parameters can be calculated from this. The S_{ij} terms are the transmission coefficients. From this quantity, gain in an active device, loss in a passive device, insertion loss, group delay, and other related parameters can be found.

An analogy to optics and reflected light can be helpful in explaining the concept of S-parameters. If a ray of light incident is projected on a piece of clear glass (as shown in Figure 2), then the incident light wave would be the equivalent of a_1 seen in Figure 1. The reflected wave would be the equivalent of b_1, the transmitted wave would be b_2, and the characteristic impedance would be that of free space (~377Ω). Therefore, the transmitted wave can be thought of as S_{21} and the reflected wave as S_{11}.

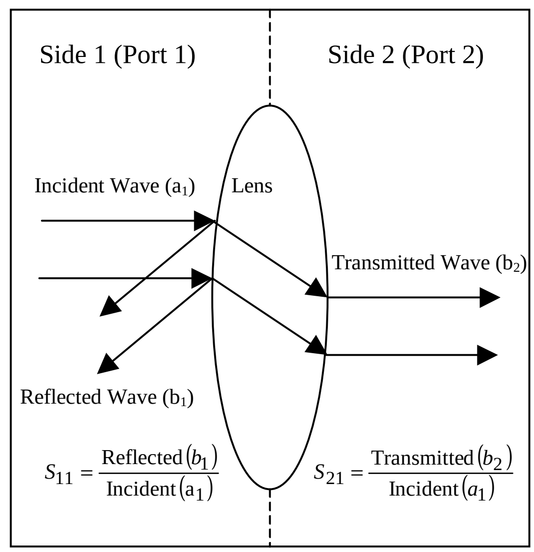

Figure 2. Optical analogy to S-parameters

3 Differential S-Parameters

When S-parameters for single-ended networks are understood, one can extend this knowledge of S-parameters to characterize differential circuits. To arrive at an understanding of differential S-parameters, a differential amplifier will first be studied. An example of a differential amplifier is shown in Figure 3.

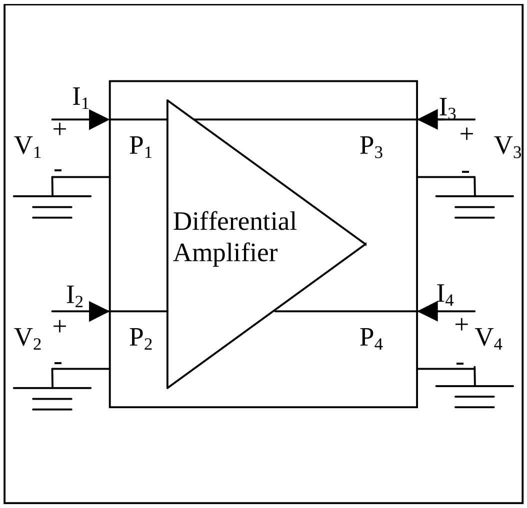

Figure 3. Example of a differential amplifier

To characterize the differential amplifier in Figure 3, each lead can be identified as a port and the differential circuit labeled as a four-port network. This approach treats the amplifier as a single-ended device. To measure the S-parameters for this single-ended approach using a two-port vector network analyzer (VNA), terminate the two unused ports with 50Ω and measure the two-port S-parameters for the two unterminated ports. Continue to terminate and measure the ports of the device in this fashion until enough information is gathered to construct the four-by-four S-parameter matrix. Assuming a good calibration for single-ended performance, the device is accurately characterized with the four-port S-parameters measured. These S-parameters do not provide much insight into the amplifier's differential (or common-mode) operation, because each port contains the differential and common-mode response. The amplifier is designed for differential operation.

A system similar to that used to describe the four transfer gains (Acc, Add, Acd, and Adc, introduced by Middlebrook [3]) is used to overcome this problem. Bockelman and others [2] introduced this system of S-parameters (called "mixed-mode"). It begins by grouping ports one and two of the amplifier in Figure 3 together (to form a differential port one) and ports three and four of the amplifier in Figure 3 together (to form a differential port two). This grouping is shown in Figure 4.

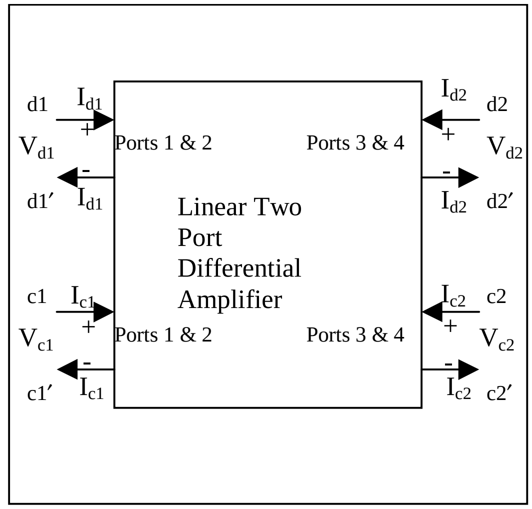

Figure 4. Example of a differential (mixed-mode) circuit

The network shown in Figure 4 is only a tool for visualizing the operation of a differential circuit. In reality, there are just two differential ports, with each port having both differential and common-mode signals.

Comparing the single-ended voltages and currents in Figure 3 with the differential voltages and currents in Figure 4, the differential and common-mode voltages and currents can be defined as:

$$v_{d1} = v_1 - v_2 \qquad v_{c1} = \frac{v_1 + v_2}{2} \quad (5)$$

$$i_{d1} = \frac{i_1 - i_2}{2} \qquad i_{c1} = i_1 + i_2 \quad (6)$$

$$v_{d2} = v_3 - v_4 \qquad v_{c2} = \frac{v_3 + v_4}{2} \quad (7)$$

$$i_{d2} = \frac{i_3 - i_4}{2} \qquad i_{c2} = i_3 + i_4 \quad (8)$$

A way can be found to convert from single-ended S-parameters to mixed-mode S-parameters with this conversion from single-ended voltages and currents to differential and common-mode voltages and currents. Before the conversion is given, a review of what has been done in the past to measure circuits differentially will be presented.

In the past, if differential measurements were desired, a balun (or hybrid coupler) would be needed, as seen in Figure 5. The problems associated with this method are (1) magnitude and phase imbalance of the baluns, (2) no way to measure mode conversion (that is, from differential to common-mode), (3) no rigorous definition of mixed-mode S-parameters, and (4) calibration of the system with the baluns is poorly defined.

Because of these problems, a carefully developed system is needed to describe the device differentially.

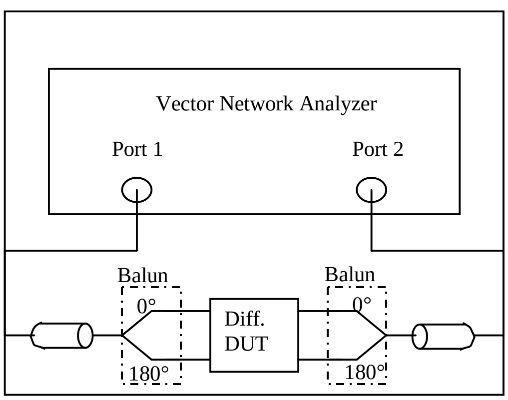

Figure 5. Sample setup to measure differential S-parameters with baluns

The need for mixed-mode S-parameters having been presented, the mixed-mode S-parameters can now be discussed. The definition for the incident and returning waves is given in equations (3 and 4) for a differential and common-mode incident and returning power wave.

In equations (9–12), v_{dn} is the differential voltage at port n, v_{cn} is the common-mode voltage at port n, i_{dn} is the differential current at port n, i_{cn} is the common-mode current at port n, Z_{dn} is the differential-mode characteristic impedance at port n ($2Z_{oo}$ for a coupled-line system), and Z_{cn} is the common-mode characteristic impedance at port n ($0.5*Z_{oe}$ for a coupled-line system).

$$a_{dn} = \frac{v_{dn} + i_{dn} Z_{dn}}{2\sqrt{\mathrm{Re}(Z_{dn})}} \quad (9)$$

$$b_{dn} = \frac{v_{dn} - i_{dn} Z_{dn}}{2\sqrt{\mathrm{Re}(Z_{dn})}} \quad (10)$$

$$a_{cn} = \frac{v_{cn} + i_{cn} Z_{nc}}{2\sqrt{\mathrm{Re}(Z_{cn})}} \quad (11)$$

$$b_{cn} = \frac{v_{cn} - i_{cn} Z_{cn}}{2\sqrt{\mathrm{Re}(Z_{cn})}} \quad (12)$$

To calculate Z_d, remember that Z = V/I; therefore, to get Z_d,, take v_d (from equation (5)) and divide it by i_d (from equation (6)). A similar method is needed to calculate Z_c. Following these steps yields

$$Z_d = 2Z_{oo} \qquad Z_c = \frac{Z_{oe}}{2} \quad (13)$$

In equation (13), Z_{oo} is the odd-mode impedance and Z_{oe} is the even-mode impedance of the system.

With the definition of the power waves in equations (9–12), the mixed-mode S-parameters can be defined as:

$$\begin{aligned} b_{d1} &= s_{dd11}a_{d1} + s_{dd12}a_{d2} + s_{dc11}a_{c1} + s_{dc12}a_{c2} \\ b_{d2} &= s_{dd21}a_{d1} + s_{dd22}a_{d2} + s_{dc21}a_{c1} + s_{dc22}a_{c2} \\ b_{c1} &= s_{cd11}a_{d1} + s_{cd12}a_{d2} + s_{cc11}a_{c1} + s_{cc12}a_{c2} \\ b_{c2} &= s_{cd21}a_{d1} + s_{cd22}a_{d2} + s_{cc21}a_{c1} + s_{cc22}a_{c2} \end{aligned} \quad (14)$$

where the following notation is used:

$$S_{ghij} = S_{(output\text{-}mode)(input\text{-}mode)(output\text{-}port)(input\text{-}port)} \quad (15)$$

which can be represented in the following format:

$$\begin{bmatrix} b_{d1} \\ b_{d2} \\ b_{c1} \\ b_{c2} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} S_{dd11} & S_{dd12} \\ S_{dd21} & S_{dd22} \end{bmatrix} & \begin{bmatrix} S_{dc11} & S_{dc12} \\ S_{dc21} & S_{dc22} \end{bmatrix} \\ \begin{bmatrix} S_{cd11} & S_{cd12} \\ S_{cd21} & S_{cd22} \end{bmatrix} & \begin{bmatrix} S_{cc11} & S_{cc12} \\ S_{cc21} & S_{cc22} \end{bmatrix} \end{bmatrix} \begin{bmatrix} a_{d1} \\ a_{d2} \\ a_{c1} \\ a_{c2} \end{bmatrix} \quad (16)$$

where S_{dd} are the differential S-parameters, S_{cc} are the common-mode S-parameters, S_{dc} are the mode conversion that occurs when the device is excited with the common-mode signal and the differential signal is measured, and S_{cd} are the mode conversion that occurs when the device is excited with a differential-mode signal and the common-mode response is measured. This mode conversion is unavoidable, because (whether intentionally or not) there is a common ground to the entire circuit or device mismatch and imbalance.

To convert from single-ended S-parameters to mixed-mode S-parameters, it is assumed that the device under test is being fed from differential input lines and that $Z_{oe} = Z_{oo} = Z_0$. The assumption of differential input lines is not limiting, as we can define the length of the lines to be arbitrarily small [2, 4]. The assumption of $Z_{oe} = Z_{oo} = Z_0$ implies that the differential input lines are not coupled. This is a valid assumption, as the lines of the VNA are coaxial and not coupled [2, 4].

Taking the definitions of v_d, v_c, i_d, and i_c from equations (5–8) and using them in equations (9–12), and taking Z_d to be $2Z_0$ (from equation (13)), the following equations result:

$$a_{d1} = \frac{a_1 - a_2}{\sqrt{2}} \quad a_{c1} = \frac{a_1 + a_2}{\sqrt{2}} \quad (17)$$

$$b_{d1} = \frac{b_1 - b_2}{\sqrt{2}} \quad b_{c1} = \frac{b_1 + b_2}{\sqrt{2}} \quad (18)$$

$$a_{d2} = \frac{a_3 - a_4}{\sqrt{2}} \quad a_{c2} = \frac{a_3 + a_4}{\sqrt{2}} \quad (19)$$

$$b_{d2} = \frac{b_3 - b_4}{\sqrt{2}} \quad b_{c2} = \frac{b_3 + b_4}{\sqrt{2}} \quad (20)$$

A convenient matrix representation of equations (17–20) is given below:

$$\begin{bmatrix} a_{d1} \\ a_{d2} \\ a_{c1} \\ a_{c2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad (21)$$

$$\begin{bmatrix} b_{d1} \\ b_{d2} \\ b_{c1} \\ b_{c2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \quad (22)$$

More compactly, it can be represented as equation (23):

$$a^{mm} = Ma^{std} \quad b^{mm} = Mb^{std} \quad (23)$$

In equation (23), the superscript “mm” represents mixed-mode and the superscript “std” represents the standard parameters; “M” is given by equation (24):

$$M = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad (24)$$

Applying the conversion from a^{std} and b^{std} to a^{mm} and b^{mm} (given in equation (23)) to the definition for single-ended S-parameters (given in equation (2)) yields the following equation:

$$S^{mm} = MS^{std} M^{-1} \quad (25)$$

It is important to follow the port-number scheme given in Figure 3. If the four ports are not numbered in this fashion, then equation (24) for “M” will not be correct. It will have to be arranged for equation (25) to work correctly. Section 3 up to this point is referenced in [2, 4].

The return loss of the MAX3950 10Gbps deserializer was measured using an Agilent 8753D vector network analyzer to demonstrate this technique. This network analyzer goes only to 6GHz, so just this data is presented. The measurement test setup is shown in Figure 6.

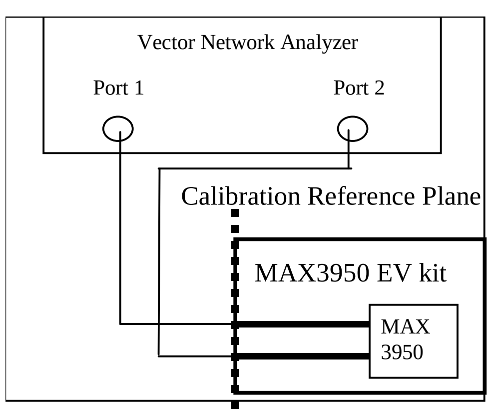

Figure 6. Measurement setup for MAX3950 return loss

A standard SOLT calibration was performed using the HP85033D 3.5mm calibration kit to obtain the measured data. Once the calibration was performed, the measurement reference plane was moved to the end of the cables, as shown in Figure 6. With no calibration kit built to measure the device on its circuit board, all S-parameters presented include the effects of the transmission line and SMA connectors to get from the VNA to the MAX3950. As a result, the actual return loss of the device will be better than that presented here.

In comparison, the single-ended return loss is presented in Figure 7.

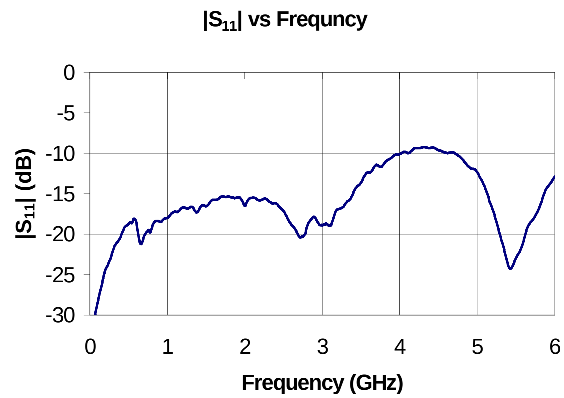

Figure 7. Single-ended return loss (S_{11}) for the MAX3950

To get a true differential return loss, apply equation (25) to the measured data. This result is given in Figure 8. To validate the conversion, the differential return loss of the MAX3950 on the MAX3950 EV kit was measured using ATN Microwave’s ATN-4000 series differential network analyzer system. The results of the differential network analyzer are also presented in Figure 8.

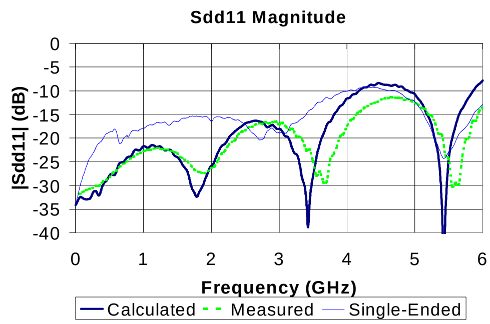

Figure 8. Single-ended and differential S-parameters of the MAX3950

It is clear that the actual differential return loss is different from the single-ended return loss. The difference between the measured data (i.e., measured with the ATN-4000 series network analyzer) and the calculated data (i.e., measured in a single-ended fashion and converted to differential) is most likely a result of two different machines with two different calibrations used for the measurement. The most accurate way to characterize the part is with the differential return loss, because the device is a differential part.

4 Calibration Issues

The return loss of the MAX3950 was presented in the previous section. But how accurate are those measurements? Like all measurements, they are only as accurate as the equipment, the methodology, and the operator are. Good test equipment and calibration standards are required to ensure the most accurate S-parameter measurements possible.

The calibration standards used depend on the measurement setup. Some possible calibration standards for the vector network analyzer are short, open, load, and through (SOLT), which are provided in the Agilent 3.5mm calibration kit or on an impedance standard substrate (ISS) like the one provided by Cascade Microtech with wafer probes. Other options include using through, reflect, line (TRL); line, reflect, match (LRM); line, reflect, reflect, match (LRRM); short, open, load, reciprocal (SOLR), or a custom-defined calibration kit. Whichever calibration scheme is used, the desired end result is the same: accurate and repeatable measurements.

An analogy of optics and light can be used to describe S-parameters. Imagine that the device under test being measured is an object behind a piece of dirty glass. To take an accurate picture of the object, the glass first needs to be cleaned. Think of calibration as cleaning or removing the glass to get the clearest possible picture.

In a typical VNA measurement system, there are cables to connect the VNA to the DUT (either through SMA connectors, wafer probes, or other connectors not mentioned here). This cable and connector have a significant effect in obscuring the measured data just like the dirty glass affecting a clear photo. Without a calibration, the response of the device and the connectors is being measured. A calibration is needed to remove the cable and connector effects.

One’s position is defined with a calibration (for example, SOLT) on the Smith chart. The open standard defines the right edge of the Smith chart, the short standard defines the left edge of the Smith chart, and the load standard defines the center of the Smith chart (see Figure 9).

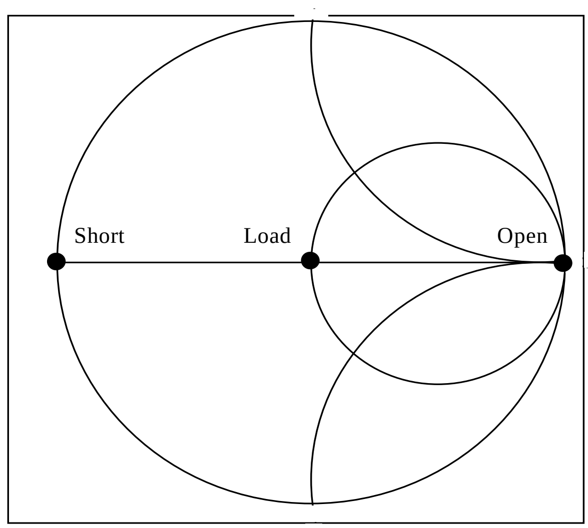

Figure 9. Smith chart

The through standard (needed only for two-port measurements) identifies the delay and the loss introduced by the cables and the connectors. Figure 10 shows a test setup with the reference planes defined before and after calibration. Returning to the analogy of optics, the reference plane is like the camera placement in relation to the object (that is, the side of the glass where the camera is placed to take the picture of the object).

In Figure 10, the transmission-line elements are everything that is between the VNA and the DUT (cables, connectors, transmission lines on printed circuit boards, matching networks, etc.). The measurement results will not accurately report the DUT's performance if all the elements between the VNA and the DUT are not correctly identified and calibrated out.

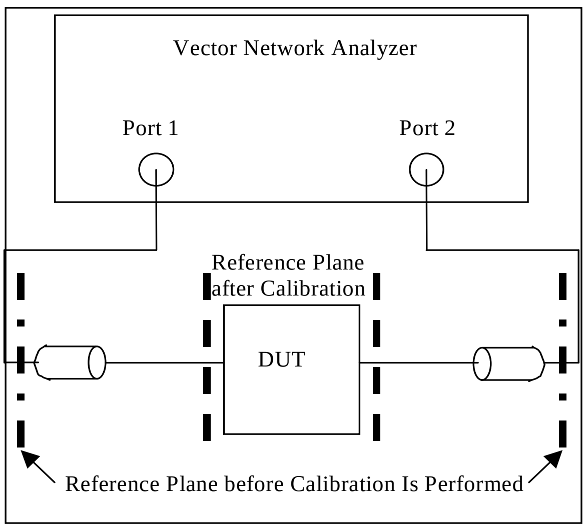

Figure 10. Sample measurement setup

The importance of accurate characterization of the elements between the DUT and the VNA, the return loss (S_{11}) of the MAX3875, is illustrated by means of measurements taken after two different calibrations were performed.

The first calibration was accomplished with the Agilent 3.5mm D calibration kit. This procedure served to calibrate out the effects of the cable connecting the printed circuit board and the vector network analyzer. (See Figure 11.)

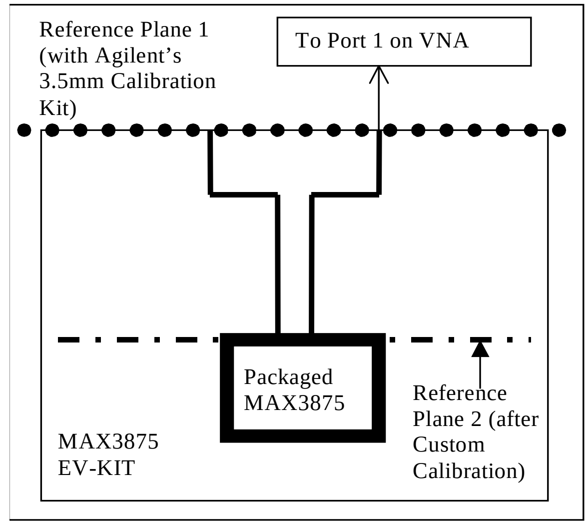

Figure 11. Different reference planes

The second calibration was performed using custom-defined SOL (short, open, load) standards. This custom calibration kit was built with short, open, and load standards placed on the board at the point where the DUT is mounted (see Figure 12). One board is manufactured for each of the standards. This allows for the removal of the effects of the transmission line on the PCB. The determination of these custom standards is outside the scope of this article, but the interested reader is referred to reference [5] on page 10. It should be noted that, since 0402 series surface-mount components were used, this calibration kit loses accuracy above approximately 4GHz [5].

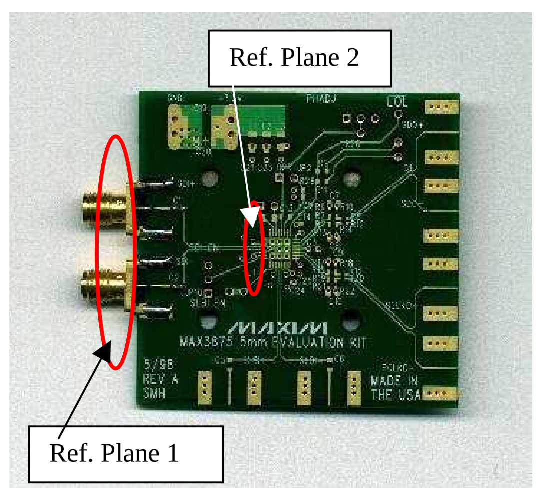

Figure 12. Calibration board

One might think that calibrating the network analyzer to the end of the cables using a standard calibration kit would be sufficient. However, as illustrated in Figure 13, the measured return loss changes if the reference plane is moved from the edge of the MAX3875 EV kit to the input of the packaged part.

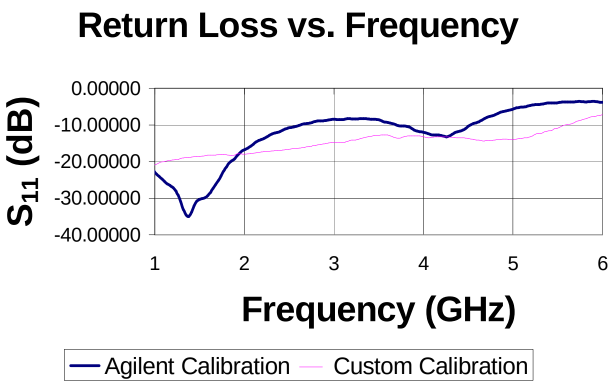

Figure 13. Return loss for the MAX3875

The reason for this difference is apparent from the physical layout of the MAX3875 EV kit (see Figure 14). Agilent's 3.5mm D calibration kit only corrects for the cable from the network analyzer to the circuit board, not to the edge of the part. In fact, after a calibration is done with this calibration kit, there is still the SMA connector on the board, the coupling capacitor on the transmission line to the MAX3875, and the transmission line from the SMA connectors to the MAX3875. All three of these items on the circuit board corrupt the accurate measurement of the return loss.

5 Summary

This application note introduced both single-ended and differential S-parameters. It also explained the need for calibration to ensure accurate microwave measurements. Section 2 suggested optics and transmitted light as an analogy for S-parameters. Calibration was seen as analogous to the cleaning of the layers of dirty glass between the camera and the object to be photographed.

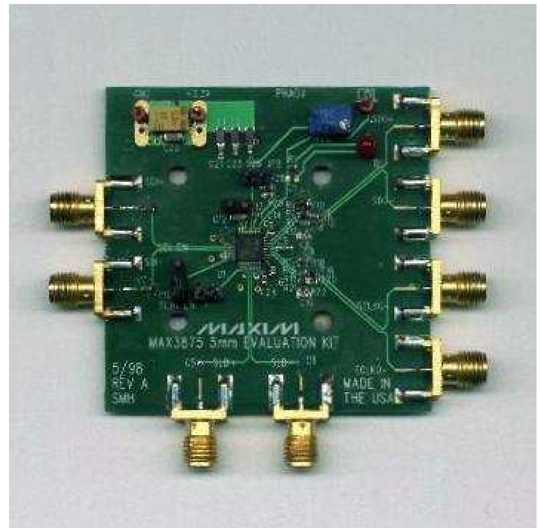

Figure 14. DUT on a printed circuit board

References

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